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Transport of a Low-Energy Beam from the Scorpius Accelerator to the Final-Focus Solenoid Title:

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# Transport of a Low-Energy Beam from the Scorpius Accelerator to the Final-Focus Solenoid

### Carl Ekdahl

#### I. INTRODUCTION

To may be required to use Scorpius for low-energy radiography of hydrodynamic experiments, perhaps as low as the 2-MeV injected beam kinetic energy. As presently designed, the downstream transport (DST) system will not work for energies less than about 4.5 MeV due to space-charge defocusing (see Fig. 1) For energies less than that the beam envelope expands to the wall before the first magnet can catch and focus it. More solenoids would have to be added to the DST lattice in order to transport a beam with KE < 4.5 MeV to the final focus.

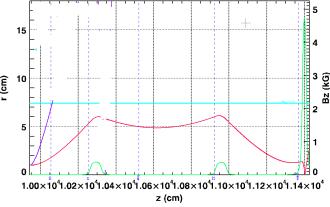


Fig. 1: Envelope-code simulation of a 5-MeV, 1.45-kA beam (red) transported from the Scorpius accelerator to the final focus with the present design. Also shown is a 2-MeV beam (purple), which expands to the beam pipe (cyan) due to space charge defocusing before it can be focused by the first solenoid (green).

The Scorpius injector produces a space-charge dominated beam with a 1.45-kA current, and this note considers DST lattice requirements for transporting beams having that current and kinetic energy less than 5-MeV. The problem is first bounded with estimates from analytic theory, and then refined with simulations using our envelope and PIC codes.

#### II. THEORY

In order to estimate the minimum number of solenoids needed, one can consider a periodic-focusing lattice for the space-charge dominated beam. Loosely following the reasoning of ref. [1, 2, 3]; using the thin lens approximation,

the beam drifts between magnets. The drifting beam envelope radius, R, is found by solving the envelope equation

$$\frac{d^2R}{dz^2} = \frac{K}{R} \tag{1}$$

 $d^2R/dz^2=K/R$ , where the generalized perveance is  $K=2\left(I_{kA}/17\right)/\left(\beta\gamma\right)^3$ , the beam kinetic energy is  $KE=\left(\gamma-1\right)m_ec^2$ , and  $\beta\gamma=\left(\gamma^2-1\right)^{1/2}$  with  $\beta=v/c$ . The perveance of a 2-MeV, 1.45-kA beam is 1.533E-3. The envelope equation can be solved by reduction to quadrature, yielding a solution for a beam launched at a waist. This gives the beam envelope divergence

$$\frac{dR}{dz} = \left(2K\ln\rho\right)^{1/2} \tag{2}$$

where  $\rho = R(z) / R_w$  and  $R_w$  is at the waist. The envelope radius can be written as a nonlinear equation

$$R(z) = (2K)^{1/2} z / G(\rho)$$
 (3)

by defining a slowly varying function

$$G(\rho) = \rho^{-1} \int_{1+}^{\rho} (\ln u)^{-1/2} du$$
 (4)

which has a maximum value  $G_{\rm max}=1.08209$  at  $\rho=2.3491$  and varies by less than 9% over the range  $1.56 < \rho < 4.57$  where G>1.

The lattice of focusing lenses for transport is designed by noting that the magnitude of divergence at the exit of one cell must be equal to the convergence entering the next cell for so-called "matched" transport with the same beam waist in every cell. This is equivalent to requiring that the magnitudes of cell entrance convergence and exit divergence are equal, which by Eq. (2) is true if the entrance and exit radii are equal. The length of such a cell with length L is found from Eq. (3) to be

$$L = 2R_0 (2K)^{-1/2} G(\rho_0)$$
 (5)

where  $R_0$  and  $\rho_0$  are at the cell boundaries. It follows that the number of solenoids to transport over a given distance  $L_0$  is

$$N = L_0 / L = (2K)^{1/2} L_0 / 2R_0 G$$
 (6)

which is minimized by maximizing  $R_0$  and G. This is achieved by setting  $G(\rho_0) = G_{\max} = 1.08209$ , thereby determining the beam waist size  $R_w = R_0 / 2.3491$ . Maximizing  $R_0$  has the disadvantage of emittance growth due spherical aberration, which scales as  $R_0' R_0^3$  [4]. Anticipating that this emittance growth will be acceptable,  $R_0 = 6$  cm is chosen for transport through the 7.3-cm inner radius beam pipe. For this choice, the number of solenoids needed for transport through the 14-m long DST is plotted vs beam energy in Fig. 2, clearly showing the  $N \propto (\beta \gamma)^{-3/2}$  scaling indicated by Eq. (6). This is a good starting point for simulations to further investigate such a system.

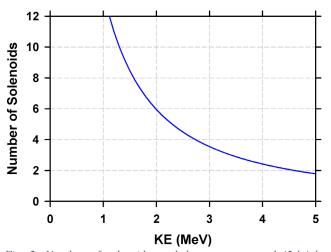


Fig. 2: Number of solenoids needed to transport at 1.45-kA beam with a 6-cm maximum envelope radius through a 14-m periodic lattice.

# III. SIMULATIONS

In the preceding section, the thin lens approximation was used to estimate the number of magnets required. However, because of the long Scorpius solenoids, fewer magnets may be needed. Envelope code simulations can refine this estimate.

However, the phase advance through long cells may cause lattice (envelope) instability. This can be investigated with PIC code simulations. Moreover, because the lattice cell length is proportional to beam size, it would be advantageous to transport a large beam. Since large beams transported by solenoids are subject to emittance growth due to spherical aberrations, this also needs exploration with PIC code simulations.

The envelope and PIC code simulations for low energy Scorpius transport are described in this section.

## A. Envelope Code Simulations

In the following, it is assumed that the Scorpius LIA can be tuned to inject a beam into the DST at the desired waist size. The XTR envelope code [5, 6] was used to simulate a 14-m long lattice similar to the Scorpius DST. The Scorpius interblock transport solenoids were used for the focusing elements. Iterating on the results of analytic estimates in the preceding section, the lattice shown in Fig. 2 was reached.

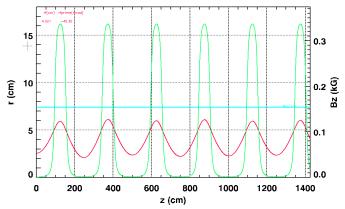


Fig. 3: Simulated periodic transport of a 2-MeV, 1-45-kA beam. Beam envelope shown in red, solenoid field on axis in green, and beam pipe wall in cyan.

The cell length for this lattice is 250 cm. Stability generally require that the phase advance per cell be less than  $\pi$ , where the phase advance is

$$\varphi = \int_{z}^{z+L} k_{\beta} dz \tag{7}$$

The total advance is shown in Fig. 4, from which one finds that the advance per cell is only about  $0.4\pi$ , comfortably below instability thresholds.

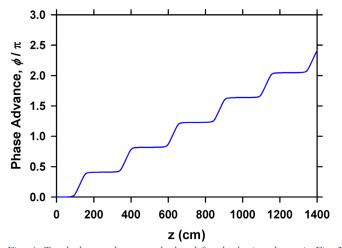


Fig. 4: Total phase advance calculated for the lattice shown in Fig. 3 including space-charge depression of beam energy.

#### B. PIC Code Simulations

Emittance growth through the 2-MeV lattice was investigated with the LSP-S PIC code, which calculates transverse dynamics of a thin beam slice [7, 6]. The slice is launched as a rigid rotor with uniform distribution as shown in Fig. 5. In this illustration the uniform density beam is color coded to show the space charge depression of kinetic energy,  $KE = (\gamma - 1)m_e c^2$ . Growth of emittance as the beam transports through the lattice is illustrated in Fig. 6.

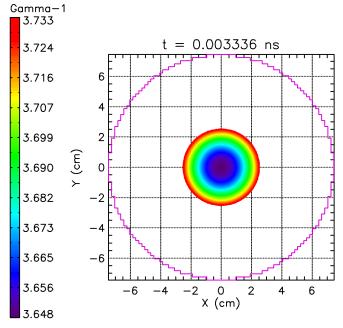


Fig. 5: Initial uniform 2-MeV beam distribution color coded to show space-charge depression of kinetic energy.

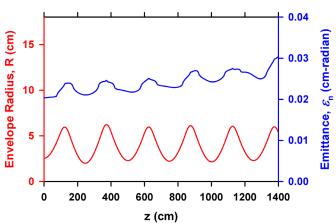


Fig. 6: Envelope radius (red) and normalized emittance (blue) simulated by the LSP-S 2D PIC code in Cartesian coordinates for the 2-MeV beam transported through the lattice shown in Fig. 3.

The emittance of the 2-MeV beam was initialized as 0.0205 cm-radian in keeping with simulations of the Scorpius injector. This grew by about 25%, as shown in Fig. 6. Emittance growth in LIAs with quasi-periodic focusing has been attributed to halo formation due to envelope (lattice)

instability, and/or to spherical aberration of the focusing solenoids [8]. Since the beam distribution near the end of this lattice shows no evidence of halo (Fig. 7), the emittance growth in this simulation is attributed to spherical aberration. This is supported by noting the beam hollowing evidenced in a slice of the initially uniform distribution (Fig. 8). Such hollowing of the profile is caused by edge focusing due to the aberration. Since the emittance growth due to spherical aberration scales as  $R_0'R_0^3$  [4], this growth could be reduced by transporting the beam at a smaller radius, although the number of solenoids in the lattice would need to be increased to accomplish this.

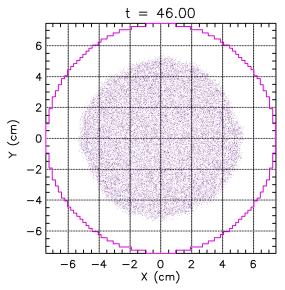


Fig. 7: 2-MeV beam distribution near the end of the 14-m long lattice.

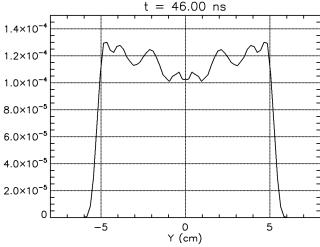


Fig. 8: Vertical slice of charge density for the 2-MeV beam distribution shown in Fig. 6, showing edge focusing due to spherical aberration.

# IV. DISCUSSION

The preceding analysis and simulations have shown that, in principle, the addition of focusing magnets to the Scorpius DST would enable transport of a low-energy beam (KE < 4.5 MeV) with only a modest increase in emittance. Once

definitive requirements for this capability have been established, the exact location and field strength of lattice focusing elements can be adjusted to support the LIA exit parameters and the optimum envelope size at the final focus entrance.

Although adding magnets to the DST enables transport of a low-energy beam, such a beam may be impractical for flash radiography, except for exceedingly thin objects. For example, our DOSECALC program [9] predicts less than 0.8Rads at one meter from a 2.0-MeV, 1.45-kA, 60-ns beam pulse focused onto a 1-mm thick Ta target (Fig. 5). The injected beam would need to be accelerated to 3.5 MeV to equal the ~4Rads produced by the Cygnus rod-pinch diode. The transport of this beam would require only one additional solenoid in the DST. However, the bremsstrahlung spectrum produced by a 3.5-MeV beam would be substantially different than that produced by a 2.0 MeV beam (Fig. 10), which is similar to the Cygnus rod-pinch spectrum [10].

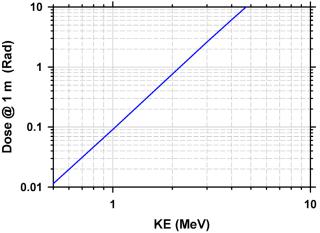


Fig. 9: Radiographic dose produced by a 1.45-kA beam focused onto a 1-mm thick Ta target. (DARHT-II final focus geometry was assumed.)

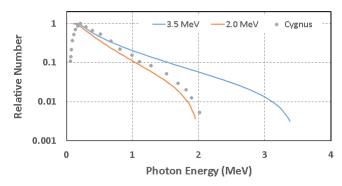


Fig. 10: Relative spectra of bremsstrahlung radiation from lowenergy electrons incident on 1-mm thick Ta targets as calculated with DOSECALC. Also shown are a few points from the measured Cygnus spectrum [10].

# V. CONCLUSION

With the addition of a few more magnets the Scorpius downstream transport section can accommodate a space-

charge dominated beam with kinetic energy less than 5 MeV. Envelope and PIC codes have been used to investigate the periodic focusing lattices that would achieve this. Transport through a lattice having the minimum number of solenoids would likely be accompanied by only a modest increase in emittance due to spherical aberration.

However, low energy Scorpius beams would not have much dose for radiography due to the low current (~1.45 kA). For example, the dose produced by such a 2-MeV beam would be less than 20% of the dose presently available from Cygnus. Acceleration to 3.5 MeV would be required to equal the Cygnus dose, which might need only a single magnet added to the DST.

#### ACKNOWLEDGMENTS

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